

Warthill Church of England Primary and Sand Hutton Church

## Introduction

The policy focuses on the four operations of addition, subtraction, multiplication and division and includes a list of the key mental maths skills that support written methods.

Each operation begins with practical methods that support conceptual understanding moving through to methods that allow children to demonstrate efficiency in procedural approaches.

It is important to emphasise that alternative methods may be more appropriate for certain calculations and that informal methods currently used successfully in schools may continue to be used as they support the raised expectations in calculation outlined in this policy.

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## Addition

## Written methods for addition

It is important that children's mental methods of calculation are practised on a regular basis and secured alongside their learning and use of written methods of addition.

The aim is that children use mental methods when appropriate, but for calculations that they cannot do in their heads they use a written method accurately and with confidence.

Children are taught and acquire secure mental methods of calculation and one written method of calculation for addition which they know they can rely on when mental methods are not appropriate.

This policy shows the possible stages of each written method for addition, each stage building towards a more refined method.

There are some key basic skills that children need to help with addition, which include:

- counting
- estimating
- recalling all number bonds to 20 and $100(7+3=10,17+3=20,70+30=100)$
- adding mentally a series of one-digit numbers $(5+8+4)$
- adding multiples of $10(60+70)$ or of $100(600+700)$ using the related addition fact, $6+7$, and their knowledge of place value
- partitioning two-digit and three-digit numbers into multiples of 100,10 and 1 in different ways $(432$ into $400+30+2$ and also into $300+120+12)$
- understanding and using addition and subtraction as inverse operations

Mathematical reasoning and problem solving is a key theme and two of the aims of National Curriculum. It is important that children's skills are broadened through their use and application in a range of contexts, these include:

- using inverses
- missing number questions
- using units of measure including money and time
- word problems
- open ended investigations


## Stage 1: Practical (combining) and adding on (increasing)

Prior to recording addition steps on a number line, children will work practically with equipment where they are combining sets of objects. As they become more confident, this practical addition of sets of objects will be mirrored on a number line so that the two are being done together and children are adding on. This will prepare them for the abstract concept of adding numbers rather than objects.

## Stage 2: Number tracks and number lines



Steps in addition can be recorded on a number line. The steps often bridge through a multiple of 10 and, this is more efficient if children know how to partition 1-digit numbers.
$8+7=15$

$48+36=84$

or


With practice, children will need to record fewer jumps

In this example, 7 has been partitioned into 2 and 5 which makes bridging through 10 more efficient

In these examples, the 6 in 36 has been partitioned into 2 and 4 which makes bridging through 10 more efficient

## Stage 3: Dienes to introduce Column Addition

To ensure the statutory final written method is grounded in understanding, this stage connects the practical equipment to the formal written method using a similar and transferrable layout.

Children are taught to regroup the ones if they make more than 10 . They then carry the new 10 to the tens column. This is consistently referred to as 'regrouping'.
As part of the written layout children show where regrouping has taken place by placing the new ten (or hundred, thousand etc) underneath the answer line of the new column.


## Stage 4: Column Method

Children should be encouraged to estimate their answers first. Regrouping is shown underneath the answer lines in the relevant
 column.

Column addition remains efficient when used with larger whole numbers or decimals, and when adding more than two numbers, once learned, the method is quick and reliable.

## Subtraction

## Written methods for Subtraction

It is important that children's mental methods of calculation are practised on a regular basis and secured alongside their learning and use of written methods of subtraction.

The aim is that children use mental methods when appropriate, but for calculations that they cannot do in their heads they use a written method accurately and with confidence.

Children are taught and acquire secure mental methods of calculation and one written method of calculation for subtraction which they know they can rely on when mental methods are not appropriate.

This policy shows the possible stages of each written method for subtraction, each stage building towards a more refined method.

There are some key basic skills that children need to help with subtraction, which include:

- counting
- estimating
- recalling all number bonds to 20 and 100 along with their inverses $(7+3=10,10-3=7,17+3=$ $20,20-3=17,70+30=100,100-30=70$ )
- subtracting multiples of $10(160-70)$ using the related subtraction fact, $16-7$, and their knowledge of place value
- partitioning two-digit and three-digit numbers into multiples of 100,10 and 1 in different ways (432 into $400+30+2$ and also into $300+120+12$ )
- understanding and using subtraction and addition as inverse operations

Mathematical reasoning and problem solving is a key theme and two of the aims of National Curriculum. It is important that children's skills are broadened through their use and application in a range of contexts, these include:

- using inverses
- missing number questions
- using units of measure including money and time
- word problems
- open ended investigations


## Stage 1: Practical (taking away)

Prior to recording subtraction steps on a number line, children will work practically with equipment where they are 'taking away' a small group from a larger set of objects. As they become more confident, this practical subtraction will be mirrored on a number line and children will be able to find the difference in order to subtract. This will prepare them for the abstract concept of subtracting numbers rather than objects.


Steps in subtraction can be recorded from right to left on a number line. The steps often bridge through a multiple of 10 and, this is more efficient if children know how to partition 1-digit numbers.
$15-7=8$

$74-27=47$

or


174-27 = 147


In this example, 7 has been partitioned into 2 and 5 which makes bridging through 10 more efficient

In these examples, 27 has been partitioned into tens and units then the 7 in 27 has been partitioned into 3 and 4 which makes bridging through 10 more efficient

Counting up (to be introduced after counting back)
Steps in subtraction can be recorded from left to right on a number line. The steps often bridge through a multiple of 10 .


When carrying out money calculations that
 involve finding change or when calculating time duration, children should use this method

With practice, children will need to record fewer jumps.
They will decide whether to count back or forwards, seeing both as 'finding the difference'. It is useful to ask children whether counting up or back is the more efficient for calculations such as $57-12$ or $86-77$.

## Stage 3: Dienes to Support Column Method

To ensure the statutory final written method is grounded in understanding, this stage connects the pract equipment to the formal written method using a similar and transferrable layout.
$72-47$


7 ones and then 4 tens are removed, leaving 25 . The 25 can be dragged to the botton to model the recording used in the written algorithm


As with this example children are taught to regroup the tens to make it possible to subtract the 7 ones.. The tens they have regrouped from is crossed out to show the new number of tens and the ten is shown the ones column. This column now has the value of 12 . This is consistently referred to as 'regrouping'.

## Stage 4: Column Method

Extend to more complex combinations such as three two-digit numbers, two three-digit numbers, and problems involving several numbers of different sizes, including decimals.

## 51 Here regrouping is shown.

543
246
317

## Multiplication

## Written methods for Multiplication

It is important that children's mental methods of calculation are practised on a regular basis and secured alongside their learning and use of written methods of multiplication.

The aim is that children use mental methods when appropriate, but for calculations that they cannot do in their heads they use a written method accurately and with confidence.

Children are taught and acquire secure mental methods of calculation and one written method of calculation for multiplication which they know they can rely on when mental methods are not appropriate.

This policy shows the possible stages of each written method for multiplication, each stage building towards a more refined method.

There are some key basic skills that children need to help with multiplication, which include:

- counting in steps
- estimating
- understanding multiplication as repeated addition
- recalling all multiplication facts to $12 \times 12$
- partitioning numbers into multiples of one hundred, ten and one
- working out products $(70 \times 5,70 \times 50,700 \times 5,700 \times 50)$ using the related fact $7 \times 5$ and their knowledge of place value
- using addition effectively to support with multiplication.
- understanding and using division and multiplication as inverse operations

Mathematical reasoning and problem solving is a key theme and two of the aims of National Curriculum. It is important that children's skills are broadened through their use and application in a range of contexts, these include:

- using inverses
- missing number questions
- using units of measure including money and time
- word problems
- open ended investigations


## Stage 1: Practical (repeated addition)

Children will work practically with equipment grouping objects to see multiplication as repeated addition. As they become more confident, this practical grouping of objects will be mirrored on a number line using the vocabulary 'lots of', 'groups of', 'how many lots', 'how many times' so that the two are being done together. This will prepare them for the abstract concept of multiplying numbers rather than objects.


This image can be expressed as:

- 2 multiplied by 5
- two, five times
- 5 groups of 2
- 5 lots of 2
- 5 jumps of 2 on a number line


## Stage 2: Practical and pictorial arrays (towards grid method)

Children use arrays to demonstrate their understanding of commutativity for multiplication facts


Children use their knowledge of known multiplication tables

This $3 \times 7$ array can also be seen as $3 \times 5$ add $3 \times 2$
$3 \times 7=21$

## Stage 3: Partitioning (Grid Method)

$4 \times 13=$


This then becomes

| x | 10 | 3 |
| :---: | :---: | :---: |
| 4 | 40 | 12 |

$40+12=52$

To ensure the statutory final written method is grounded in understanding, this stage connects the practical equipment to the formal written method using a similar and transferrable layout.


## Stage 5: Long (column)

$24 \times 32=768$


$1245 \times 13$
1245
$\times \quad 13$$\quad \begin{aligned} & \text { It } \\ & \text { is } \\ & \text { children understand that } \\ & \text { this } 0 \text { is here as a place } \\ & \text { holder because we are } \\ & \text { multiplying } 10 \times 5 \text { not } 1 \times\end{aligned}$
$\begin{array}{r}1243 \\ \times 3735 \\ 12450 \\ \hline 16185\end{array}$

It is important that children understand that this 0 is here as a place multiplying $10 \times 5$ not 1 x 5.

## Division

## Written methods for Division

It is important that children's mental methods of calculation are practised on a regular basis and secured alongside their learning and use of written methods of division.

The aim is that children use mental methods when appropriate, but for calculations that they cannot do in their heads they use a written method accurately and with confidence.
Children are taught and acquire secure mental methods of calculation and one written method of calculation for division which they know they can rely on when mental methods are not appropriate.

This policy shows the possible stages of each written method for division, each stage building towards a more refined method.

There are some key basic skills that children need to help with subtraction, which include:

- counting
- estimating
- understanding division as repeated subtraction
- partitioning two-digit and three-digit numbers into multiples of 100,10 and 1 in different ways ( 432 into $400+30+2$ and also into $300+120+12$ )
- recalling multiplication and division facts to $12 \times 12$
- recognising multiples of one-digit numbers and dividing multiples of 10 or 100 by a single-digit number using their knowledge of division facts and place value
- knowing how to find a remainder working mentally, for example, find the remainder when 48 is divided by 5
- understanding and using division and multiplication as inverse operations

Using and applying is a key theme and one of the aims of National Curriculum and before children move onto the next stage in written calculation it is important that their skills are broadened through their use and application in a range of contexts, these include:

- using inverse
- missing box questions
- using units of measure including money and time
- word problems
- open ended investigations


12 cakes are shared equally between 3 people.

## Stage 2: Number lines

Children will move from sharing objects practically to grouping them, this will be mirrored on a number line, working from right to left so that they see division as repeated subtraction. This will prepare them for the abstract concept of dividing numbers rather than objects.


Each cake box holds 3 cakes, if I have 12 cakes, how many cake boxes will I need?


How many times can I subtract 3 from 12?

Using their knowledge of the inverse relationship between multiplication and division, children can use their multiplication tables when grouping on a number line, working from left to right.


How many groups of 3 are there in 12 ?

First without and then with remainders and ensuring that divisors offer an appropriate level of challenge.

## Stage 4: Using arrays - Making Links to Multiplication



Grouping: If we put 4 apples in each bag how many bags will be full? How many will be in the bag that is not full?
How many bags will we need?

Sharing the apples: How can we share the two apples that are left between four children? How many apples will each child get?
How many apples would each child have?
8


Divided (56) $\div$ divisor (7) $=$ Quotient (8)

## Stage 5: Using Arrays to Support Short Division



Using the principles of arrays linked to place value $363 \div 3$.
Each part of the number is grouped or shared into the divisor. Explaining the recording of the division as;


This then becomes more complex when exchange is needed as complete groups of the divisor cannot be made e.g.


Then becomes


Recorded as
115

Stage 5: Short division
$560 \div 24=23$ r8 $432 \div 15=28$ r12


$(12 \div 15=0.8)$
remainder as a decimal
$1 5 \longdiv { 2 8 3 \frac { 4 } { 5 } ^ { 4 } }$
$30 \downarrow$
132
$\frac{120}{12}$
12
( $0.8=\frac{4}{5}$ )
remainder as a fraction

With long division, there is the
opportunity to teach an expanded
method first (ie chunking)

