

Warthill Church of England Primary and Sand Hutton Church


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## Introduction

The policy focuses on the four operations of addition, subtraction, multiplication and division and includes a list of the key mental maths skills that support written methods.

For each operation, there are four stages, starting with the practical methods that support conceptual understanding moving through to methods that allow children to demonstrate efficiency in procedural approaches.

It is important to emphasise that alternative methods may be more appropriate for certain calculations and that informal methods currently used successfully in schools may continue to be used as they support the raised expectations in calculation outlined in this policy.

## Addition

## Written methods for addition

It is important that children's mental methods of calculation are practised on a regular basis and secured alongside their learning and use of written methods of addition.

The aim is that children use mental methods when appropriate, but for calculations that they cannot do in their heads they use a written method accurately and with confidence.

Children are taught and acquire secure mental methods of calculation and one written method of calculation for addition which they know they can rely on when mental methods are not appropriate.

This policy shows the possible stages of each written method for addition, each stage building towards a more refined method.

There are some key basic skills that children need to help with addition, which include:

- counting
- estimating
- recalling all addition pairs to 10,20 and $100(7+3=10,17+3=20,70+30=100)$
- knowing number facts to $10(6+2=8)$
- adding mentally a series of one-digit numbers $(5+8+4)$
- adding multiples of $10(60+70)$ or of $100(600+700)$ using the related addition fact, $6+$ 7, and their knowledge of place value
- partitioning two-digit and three-digit numbers into multiples of 100,10 and 1 in different ways ( 432 into $400+30+2$ and also into $300+120+12$ )
- understanding and using addition and subtraction as inverse operations

Using and applying is a key theme and one of the aims of National Curriculum and before children move onto the next stage in written calculation it is important that their skills are broadened through their use and application in a range of contexts, these include:

- using inverse
- missing box questions
- using units of measure including money and time
- word problems
- open ended investigations


## Stage 1: Practical (combining) and adding on (increasing)

Prior to recording addition steps on a number line, children will work practically with equipment where they are combining sets of objects. As they become more confident, this practical addition of sets of objects will be mirrored on a number line so that the two are being done together and children are adding on. This will prepare them for the abstract concept of adding numbers rather than objects.

## Stage 2: Number tracks and number lines



Steps in addition can be recorded on a number line. The steps often bridge through a multiple of 10 and, this is more efficient if children know how to partition 1-digit numbers.

$48+36=84$

or


With practice, children will need to record fewer jumps

In this example, 7 has been partitioned into 2 and 5 which makes bridging through 10 more efficient

In these examples, the 6 in 36 has been partitioned into 2 and 4 which makes bridging through 10 more efficient

## Stage 3: Partitioning (expanded columnar method)

Partition both numbers into tens and units or hundreds, tens and units (using a grid makes this easier)..
$48+36=84$

$148+36=184$


This builds on children's mental maths skills of partitioning and
recombining $40+30=70$
$8+6=14$
$48+36=84$

## Stage 4: Efficient (column method)



Chidren should be encouraged to estimate their answers first

Column addition remains efficient when used with larger whole numbers or decimals, and when adding more than two numbers, once learned, the method is quick and reliable.

## Subtraction

## Written methods for Subtraction

It is important that children's mental methods of calculation are practised on a regular basis and secured alongside their learning and use of written methods of subtraction.

The aim is that children use mental methods when appropriate, but for calculations that they cannot do in their heads they use a written method accurately and with confidence.

Children are taught and acquire secure mental methods of calculation and one written method of calculation for subtraction which they know they can rely on when mental methods are not appropriate.

This policy shows the possible stages of each written method for subtraction, each stage building towards a more refined method.

There are some key basic skills that children need to help with subtraction, which include:

- counting
- estimating
- recalling all addition pairs to 10,20 and 100 along with their inverses $(7+3=10,10-3=$ $7,17+3=20,20-3=17,70+30=100,100-30=70$ )
- knowing number facts to 10 and their inverses ( $6+2=8,8-2=6$ )
- subtracting multiples of $10(160-70)$ using the related subtraction fact, 16-7, and their knowledge of place value
- partitioning two-digit and three-digit numbers into multiples of 100,10 and 1 in different ways ( 432 into $400+30+2$ and also into $300+120+12$ )
- understanding and using subtraction and addition as inverse operations

Using and applying is a key theme and one of the aims of National Curriculum and before children move onto the next stage in written calculation it is important that their skills are broadened through their use and application in a range of contexts, these include:

- using inverse
- missing box questions
- using units of measure including money and time
- word problems
- open ended investigations


## Stage 1: Practical (taking away)

Prior to recording subtraction steps on a number line, children will work practically with equipment where they are 'taking away' a small group from a larger set of objects. As they become more confident, this practical subtraction will be mirrored on a number line so that the two are being done together. This will prepare them for the abstract concept of subtracting numbers rather than objects.

## Stage 2: Number tracks and number lines



Counting back (to be introduced before counting up)
Steps in subtraction can be recorded from right to left on a number line. The steps often bridge through a multiple of 10 and, this is more efficient if children know how to partition 1-digit numbers.
$15-7=8$

or


174-27 = 147


With practice, children will need to record fewer jumps.

In this example, 7 has been partitioned into 2 and 5 which makes bridging through 10 more efficient

In these examples, 27 has been partitioned into tens and units then the 7 in 27 has been partitioned into 3 and 4 which makes bridging through 10 more efficient

Counting up (to be introduced after counting back)
Steps in subtraction can be recorded from left to right on a number line. The steps often bridge through a multiple of 10 .


When carrying out money calculations that involve finding change or when calculating time duration, children should use this method


With practice, children will need to record fewer jumps.
They will decide whether to count back or forwards, seeing both as 'finding the difference'. It is useful to ask children whether counting up or back is the more efficient for calculations such as $57-12$ or $86-77$.

## Stage 3: Partitioning (expanded columnar method)

Partition both numbers into tens and units or hundreds, tens and units (using a grid makes this easier).


## Stage 4: Efficient (column method)



Children should be encouraged to
estimate their answers first

Column subtraction remains efficient when used with larger whole numbers or decimals, once learned, the method is quick and reliable.

## Multiplication

## Written methods for Multiplication

It is important that children's mental methods of calculation are practised on a regular basis and secured alongside their learning and use of written methods of multiplication.

The aim is that children use mental methods when appropriate, but for calculations that they cannot do in their heads they use a written method accurately and with confidence.

Children are taught and acquire secure mental methods of calculation and one written method of calculation for multiplication which they know they can rely on when mental methods are not appropriate.

This policy shows the possible stages of each written method for multiplication, each stage building towards a more refined method.

There are some key basic skills that children need to help with multiplication, which include:

- counting
- estimating
- understanding multiplication as repeated addition
- recalling all multiplication facts to $12 \times 12$
- partitioning numbers into multiples of one hundred, ten and one
- working out products ( $70 \times 5,70 \times 50,700 \times 5,700 \times 50$ ) using the related fact $7 \times 5$ and their knowledge of place value
- adding two or more single-digit numbers mentally
- adding multiples of $10(60+70)$ or of $100(600+700)$ using the related addition fact, $6+7$, and their knowledge of place value
- adding combinations of whole numbers
- understanding and using division and multiplication as inverse operations

Using and applying is a key theme and one of the aims of National Curriculum and before children move onto the next stage in written calculation it is important that their skills are broadened through their use and application in a range of contexts, these include:

- using inverse
- missing box questions
- using units of measure including money and time
- word problems
- open ended investigations


## Stage 1: Practical (repeated addition)

Children will work practically with equipment grouping objects to see multiplication as repeated addition. As they become more confident, this practical grouping of objects will be mirrored on a number line using the vocabulary 'lots of', 'groups of', 'how many lots', 'how many times' so that the two are being done together. This will prepare them for the abstract concept of multiplying numbers rather than objects.


This image can be expressed as:

- 2 multiplied by 5
- two, five times
- 5 groups of 2
- 5 lots of 2
- 5 jumps of 2 on a number line


## Stage 2: Practical and pictorial arrays (towards grid method)

Children use arrays to demonstrate their understanding of commutativity for multiplication facts


Children use their knowledge of known multiplication tables

This $3 \times 7$ array can also be seen as $3 \times 5$ add $3 \times 2$
$3 \times 7=21$

## Stage 3: Partitioning (grid method)

$24 \times 3=72$

| $x$ | 20 | 4 |  |
| :---: | :---: | :---: | :---: |
| 3 | 60 | 12 | 72 |

$24 \times 32=768$

| $x$ | 20 | 4 |  |
| :---: | :---: | :---: | :---: |
| 30 | 600 | 120 | 720 |
| 2 | 40 | 8 | 48 |
|  |  |  | 768 |

Stage 4: Short (column)

| $24 \times 3=72$ | $1241 \times 3=3723$ |
| ---: | ---: |
| 24 | 1241 |
| $\times \quad 3$ | $\times \quad 3$ |
| 72 | 3723 |
| 1 |  |

Stage 5: Long (column)
$24 \times 32=768$
24
$\begin{array}{r}\times 32 \\ \hline 48\end{array}$
$\begin{array}{r}720 \\ \hline 765\end{array}$
$1245 \times 13$
1245
1213
$\times \quad 3735$
$\begin{array}{r}12450 \\ \hline 16185\end{array}$

In the examples given, it is also correct to multiply starting with the tens digit (ie multiplying by the most significant digit first)

## Division

## Written methods for Division

It is important that children's mental methods of calculation are practised on a regular basis and secured alongside their learning and use of written methods of division.

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Children are taught and acquire secure mental methods of calculation and one written method of calculation for division which they know they can rely on when mental methods are not appropriate.

This policy shows the possible stages of each written method for division, each stage building towards a more refined method.

There are some key basic skills that children need to help with subtraction, which include:

- counting
- estimating
- understanding division as repeated subtraction
- partitioning two-digit and three-digit numbers into multiples of 100,10 and 1 in different ways ( 432 into $400+30+2$ and also into $300+120+12$ )
- recalling multiplication and division facts to $12 \times 12$
- recognising multiples of one-digit numbers and dividing multiples of 10 or 100 by a singledigit number using their knowledge of division facts and place value
- knowing how to find a remainder working mentally, for example, find the remainder when 48 is divided by 5
- understanding and using division and multiplication as inverse operations

Using and applying is a key theme and one of the aims of National Curriculum and before children move onto the next stage in written calculation it is important that their skills are broadened through their use and application in a range of contexts, these include:

- using inverse
- missing box questions
- using units of measure including money and time
- word problems
- open ended investigations


## Stage 1: Practical (sharing)

Children will work practically with equipment sharing objects one to one.


12 cakes are shared equally between 3 people.

## Stage 2: Number lines (grouping)

Children will move from sharing objects practically to grouping them, this will be mirrored on a number line, working from right to left so that they see division as repeated subtraction. This will prepare them for the abstract concept of dividing numbers rather than objects.


Each cake box holds 3 cakes, if I have 12 cakes, how many cake boxes will I need?


How many times can I subtract 3 from 12?

Using their knowledge of the inverse relationship between multiplication and division, children can use their multiplication tables when grouping on a number line, working from left to right.


How many groups of 3 are there in 12 ?

First without and then with remainders and ensuring that divisors offer an appropriate level of challenge.

## Stage 3: Short division

$372 \div 3=124 \quad 432 \div 15=28$ r12
124
$3 \longdiv { 3 7 2 }$

15 | $28_{\mathrm{rl2}}$ |
| :---: |

$1 5 \longdiv { 4 3 2 }$
remainder as a fraction
$1 5 \longdiv { 4 3 2 }$
$\begin{aligned} & \text { remainder as a decimal }\end{aligned}$

## Stage 4: Long division

$560 \div 24=23$ r8 $432 \div 15=28$ r12


| 28.8 | 28 |
| :---: | :---: |
| $1 5 \longdiv { 4 3 2 . 0 }$ | $1 5 \longdiv { 4 3 2 }$ |
| 30】 | 30 $\downarrow$ |
| 132 | 132 |
| 120 | 120 |
| 120 | 12 |
| 120 | 12 |
| 0 |  |
| $(12 \div 15=0.8)$ | $\left(0.8{ }^{\frac{4}{5}}\right.$ ) |
| remainder as a decimal | remainder as atr |

With long division, there is the
opportunity to teach an expanded
method first (ie chunking)

